Phenomeological Determination of the Orbital Angular Momentum

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Outline

1. Proton structure status
2. Modeling the gluon asymmetry
   a. Physical constraints
   b. DGLAP evolution
3. Results for the gluon asymmetry
4. Implications for orbital dynamics
5. Phenomenology/conclusions
Proton Structure

- Quark spin (helicity) $\Delta \Sigma \approx 0.30 \pm 0.03$
- Transverse components $\Delta_T \Sigma$
- Gluon spin $\Delta G$
- Orbital motion $L_z$ (quark and gluon)
- $J_z$ sum rule: $\frac{1}{2} = \frac{1}{2} \Delta \Sigma + \Delta G + L_z$
Asymmetry Model Approach

• Model the gluon asymmetry
  \[ A(x,t) \equiv \Delta G(x,t)/G(x,t) \] where \( t \) is the evolution variable
  \[ t = \ln[\alpha_s^{LO}(Q_0^2)]/\ln[\alpha_s^{LO}(Q^2)]. \]

• Use the models for the asymmetry and the data on \( G(x,t) \) to extract \( \Delta G \).

• Combine data on \( \Delta \Sigma \) with the \( J_z \) SR to give insight on the relative size of \( L_z \).

• The models of \( A(x,t) \) give a range of possible values of \( L_z \).
Motivation

• Uses the gluon degrees of freedom to access $L_z$, complementary to Skyrme or chiral quark models

• Provides a theoretical basis for determining $\Delta G$ and $\Delta G/G$ instead of $\chi^2$ fits to data (which are still limited in kinematic range)

• Can be combined with other approaches for a cross check of results
Definitions

Gluon asymmetry: \( A(x,t) \equiv \Delta G(x,t)/G(x,t) \)

Split \( A(x,t) \) into \( t \)-dependent and \( t \)-independent parts:

\[
A(x,t) = A_0^\varepsilon(x) + \varepsilon(x,t)
\]

where \( A_0^\varepsilon(x) \equiv \left[ \partial \Delta G/\partial t \right]/\left[ \partial G/\partial t \right] \) is calculable via DGLAP evolution. Thus,

\[
\Delta G(x,t) = A_0^\varepsilon(x) \cdot G(x,t) + \Delta g_{\varepsilon}(x)
\]

where \( \Delta g_{\varepsilon}(x) = \varepsilon \cdot G \) accounts for the difference in \( \Delta G \) and \( G \) evolution at large \( t \).
Calculating the asymmetry

Choose a suitable model for $\Delta g_\varepsilon$ and use the definition of $A_0^\varepsilon(x)$ to determine the asymmetry.

$$A_0^\varepsilon = \left[ \Delta P_{Gq} \otimes \Delta q + \Delta P_{GG} \otimes (A_0^\varepsilon \bullet G + \Delta g_\varepsilon) \right] / \left[ P_{Gq} \otimes q + P_{GG} \otimes G \right]$$

Due to zeros in the denominator, the equation is transformed into.

$$A_0^\varepsilon \bullet \left[ P_{Gq} \otimes q + P_{GG} \otimes G \right] - \Delta P_{GG} \otimes [A_0^\varepsilon \bullet G] = \left[ \Delta P_{Gq} \otimes \Delta q + \Delta P_{GG} \otimes (\Delta g_\varepsilon) \right]$$
Modeling the gluon asymmetry

Physical constraints on $A_0^\varepsilon$

- **Endpoints**: $A_0^\varepsilon(0) = 0$, $A_0^\varepsilon(1) = 1$
- **Positivity**: $A_0^\varepsilon(x) \leq 1$ (all $x$)
- **Monotonicity**

To satisfy these assume $A_0^\varepsilon$ has the form

$$A_0^\varepsilon \equiv Ax^\alpha - (B - 1)x^\beta + (B - A)x^{\beta+1}$$

and generate Ansätze for $\Delta g^\varepsilon$: (positivity)

$$-0.25 \leq \int_0^1 \Delta g^\varepsilon dx \leq 0.25$$
Models of $\Delta g_\varepsilon$

• Using integral constraints to satisfy positivity, assume $\Delta g_\varepsilon$ has a form:
  - $\Delta g_\varepsilon = \pm N x^a (1-x)^b$, where the normalization is chosen to model different integral values
  - one model assumes negative correction at small $x$ and positive at large $x$:
    $\Delta g_\varepsilon = x (x-0.25) (1-x)^5$ with $<\Delta g_\varepsilon> = 0$
Asymmetry results

Asymmetry extremes as a function of $x$

\[ A_{0}^{\varepsilon} \]

Middle curve is for $\Delta g_{\varepsilon} = 0$, all $x$.

These are not Constrained by data.
Start with $J_z$ sum rule:

$$\frac{1}{2} = \frac{1}{2} \Delta \Sigma + \Delta G + L_z$$

$$\approx 0.15 + (A_0^\varepsilon(x) \cdot G + \Delta g_{\varepsilon}) + L_z$$

$$\Rightarrow L_z \approx 0.35 - \langle (A_0^\varepsilon(x) \cdot G + \Delta g_{\varepsilon}) \rangle \pm 0.04$$

Evolution:

$$\frac{\partial L_z}{\partial t} \approx - A_0^\varepsilon(x) \left[ \frac{\partial G}{\partial t} \right] \text{ at LO & NLO}$$
Constraints as a function of $\Delta g_\varepsilon$

The range of $A_0^\varepsilon$ is near linear in $x$ and satisfies all physical constraints. These are not constrained by data.

The models of $\Delta g_\varepsilon$ giving these asymmetries leads to constraints on $\Delta G$ and $L_z$

Values of $\Delta g_\varepsilon$ satisfying physical constraints:
- $-0.25 \leq \langle \Delta g_\varepsilon \rangle \leq 0.25$

Constraint on $\Delta G$: without data constraints
- $-0.09 \leq \int_0^1 \Delta G \ dx \leq 0.59 \pm 0.04$

Constraint on $L_z$:
- $-0.16 \leq \langle L_z \rangle \leq 0.44 \pm 0.04$
Results with data constraints

• The three models of $A_0^\varepsilon$ that best satisfy the limited existing data (within one sigma, without the large negative result from charm production) are:

<table>
<thead>
<tr>
<th>$\Delta g_\varepsilon$</th>
<th>$\langle \Delta g_\varepsilon \rangle$</th>
<th>$\langle \Delta G \rangle$</th>
<th>$L_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-2(1-x)^7$</td>
<td>-0.25</td>
<td>0.18</td>
<td>0.17</td>
</tr>
<tr>
<td>$-90x^2(1-x)^7$</td>
<td>-0.25</td>
<td>0.25</td>
<td>0.10</td>
</tr>
<tr>
<td>$-9x(1-x)^7$</td>
<td>-0.125</td>
<td>0.25</td>
<td>0.10</td>
</tr>
</tbody>
</table>

• The corresponding constraints on $\Delta G$ and $L_z$ are:
  • $0.18 \leq \int_0^1 \Delta G \, dx \leq 0.25 \pm 0.04$
  • $0.10 \leq L_z \leq 0.17 \pm 0.04$
Asymmetry models as a function of $x$ that are within $1\sigma$ of existing data from HERMES and COMPASS. These are constrained by data.
Asymmetry results

Asymmetry models
as a function of $x$
that are within $1\sigma$ of existing data from HERMES and COMPASS.
These are constrained by data.

Note that the $\varepsilon$ term
$=\Delta g_\varepsilon \cdot G$ must be
subtracted from the curves to match data.
Asymmetry models as a function of $x$ that are within $2\sigma \ A_0^\varepsilon$ of existing data from HERMES and COMPASS.

These are somewhat constrained by data.

The blue curve gives $\langle \Delta G \rangle = 0$ and $L_z = 0.35$ consistent with CQPM.
Other asymmetry models

• Other models include modifying $A_0^\varepsilon$ with an additional small correction, $\delta a_0$, independent of $\Delta g^\varepsilon$, which truncates at poles of the DGLAP calculation: $\delta a_0 = 0$ for $x > x_c$ where $x_c$ is at the pole (i.e. small $x \approx 0.30$) – tests the stability of the non-linear equations used to calculate the asymmetry

• These give the unconstrained limits:

$$0.00 < L_z < 0.18$$
Although the $J_z = \frac{1}{2}$ sum rule holds strictly for the integrals of the distributions, assume as a first approximation that:

$$J_z = \frac{1}{2} \approx \frac{1}{2} \Delta \Sigma(x) + \Delta G(x) + L_z(x)$$

Using $\Delta \Sigma(x)$ based on CTEQ5 and the GGR polarized distributions and the models of $\Delta G(x)$ generated here, we get a general behavior of the $x$-dependence of the orbital angular momentum of the constituents.
$L_z(x)$ with $<\Delta g_\varepsilon> = 0$ model

Plot of $L_z(x)$ vs $x$. $L_z(x)$

Small-$x$ constituents have negative OAM &
Large-$x$ constituents have positive OAM
Conclusions

1. The gluon asymmetry $\Delta G/G$ has been modeled to satisfy theoretical constraints.

2. The LO asymmetry models cluster in a range around the line $A_0^\varepsilon = x$. NLO calculations add about 10% to the asymmetry at small $x$.

3. Without data constraints:

   \[-0.09 \leq <\Delta G> \leq 0.59 \pm 0.04\]

   \[-0.16 \leq <L_z> \leq 0.44 \pm 0.04\]
Conclusions continued

4. With 1σ data constraints:
\[ 0.18 \leq \langle \Delta G \rangle \leq 0.25 \pm 0.04 \]
\[ 0.10 \leq \langle L_z \rangle \leq 0.17 \pm 0.04 \]

5. Two σ constraints allow models that give larger \( L_z \), more consistent with Covariant QPMs

6. Our results for \( L_z \) are consistent with ChQMs and provide an alternative way of calculating the OAM using gluon degrees of freedom

7. Modeling the OAM x-dependence using the \( J_z \) sum rule as a 1st approximation gives negative OAM at small-x and positive at large-x
Final conclusion

8. Measurements of $\Delta G/G$ (extension of present experiments) and $\Delta G$ alone (jet production and prompt photon production) over a wide kinematic range is very important in narrowing the allowable models of the gluon asymmetry and determination of the OAM of the nucleon constituents.

9. Determining transversity properties of the proton can add additional valuable information on the orbital angular momentum of its constituents.