Abstract: A quasi particle model for the diquark has been suggested in an analogy with the quasi particle in the crystal lattice. The magnetic moments of the exotics and the non exotic member of Baryon antidecuplet have been estimated considering the diquark-diquark- antiquark picture for the baryons. The mass of the baryon antidecuplet have also been estimated. The results are found to be in agreement with the existing theoretical and experimental results.

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Keywords: Quasi Particles; Diquarks; Magnetic Moments; Exotics; Baryon Antidecuplet.
Introduction

The magnetic moment is an important quantum number for studying the internal structure of a particle as gyromagnetic ratio give important information regarding the structure of the particle. At the theoretical level the baryon antidecuplet has been represented as the third rotational state excitation in the context of the soliton model and Soliton picture of baryons suggested existence of a baryon antidecuplet with spin $\frac{3}{2}$ as rotational excitation in addition to low lying SU$_f$(3) octet and a decuplet [1]. It is interesting to identify the non exotic members of the baryon anti decuplet as the baryon resonances. The magnetic moments of the baryons and the exotics has been investigated by a number of authors. Liu et al [2] have studied the magnetic moments of the pentaquarks in the context of different model and observed that the different model yields different values. It has been suggested that Jaffe et al [3] that the exotics like pentaquarks can be described as the diquark-diquark-antiquark system. The baryon resonances are exotic in the sense that they cannot be described as the three quark system and can be fitted as the pentaquark baryons to the baryon antidecuplet [4] although some authors [5] have argued that the such identification is somewhat arbitrary. Polyakov et al [6] have indicated that the corners of the antidecuplets are exotics and all other low lying states are rotational excitation of the same classical object-the soliton. Jaffe et al [3] have indicated that the Roper resonance and $P_{11}$ state can be fitted to the SU(3)$_f$ $\overline{TU} \oplus 8_f$ scheme. Zhu [7] has investigated pentaquark and dibaryons in the framework of diquark model and pointed that the diquarks are like antiquark in many ways which hints at the deep connection between pentaquark and dibaryons. Matheus et al [8] have estimated $\Theta^+$ and $N^*$ mass in the framework of QCD sum rule using $[uD]^2\overline{s}$ and $[ud]^2\overline{d}$. They have indicated that their results are compatible with the interpretation of $[ud]^2\overline{d}$ as ropper resonance as suggested by Jaffe et al[3].

In the present work we have studied the magnetic moment of the baryon antidecuplet considering the diquark-diquark-anti quark picture for the baryons. We have assumed that two highly correlated scalar diquark resembling quasi particle and an antiquark constitutes
antidecuplet baryons. Recently we have suggested a quasi particle picture of diquark. We have applied the model to investigate the magnetic moment. The mass of the anti decuplet baryons has been computed in the frame work of the aforesaid quasi particle diquark model. The spin contribution of the diquarks to the mass $\Delta - N$ has been computed. The difference in their mass has been attributed to the contribution of mass difference of the scalar and axial vector diquarks.

**Quasi Particle Picture of Diquark:-**

It has been suggested that at low energy the quark dynamics can be revisited in the light of the new results of baryons and exotic spectroscopy. The deeply bound diquark correlation is one of the important candidate by which the baryon spectroscopy as well as the exotic states can be described. The role of diquarks in baryon spectroscopy has been discussed by a number of authors [9]. A number of works have been done towards the understanding of the structure of diquark. The possibility of forming quark-quark and quark-antiquark system by Instanton Induced Interaction(III) have been developed by Shuryak [9] and Schafer et.al [9]. Betman et.al.[10] have investigated formation of bound state of quark-quark or quark-antiquark systems due to instanton induced interaction and predicted that such bound state is formed inside the hadron as a bubble of the size of the instanton radius. We have suggested a model for diquark in which two quarks bound together forming a quasi particle in an analogy with an electron in crystal lattice [11]. It is well-known that the quasi particles are particle like entity arising in some system of interacting particles. They are low lying excited state possessing the energy very close to the ground state and the properties of the system can be obtained by investigating the properties of the quasi particles to a considerable extent. Such states are often observed in the usual condense matter physics. A crystal electron is subjected to two type of forces namely the effect of crystal field ($\nabla V$) and an external force (F) which accelerates the electron [11]. Under the influence of the two forces, an electron in a crystal behaves like a quasi particle whose effective mass $m^*$ reflects the inertia of electrons.
which are already in a crystal field such that:

\[ m^* \frac{dv}{dt} = F \]  

(1)

and the bare electrons (with normal mass) are affected by the lattice force \(-\nabla V\), and the external force \(F\) so that:

\[ m \frac{dv}{dt} = F - \frac{dV}{dx} \]  

(2)

Hence the ratio of the normal mass \((m)\) to the effective mass \((m^*)\) can expressed as:

\[ \frac{m}{m^*} = 1 - \frac{1}{F} [\frac{\delta V}{\delta x}] \]  

(3)

We have proposed a similar type of picture for diquark \([ud]_0\) as a quasi particle inside a baryon. It is now well understood experimentally that the hadrons are complicated system. For example the complete description of proton includes with virtual sea, gluons, ss in addition to the valance quarks. So situation may be resembled to the situation of an electron in the crystal lattice. We assume that the diquark is a fundamental constituent and an independent body which is under the influence of two type of forces inside the hadron which simulates the many body interaction of the complicated structure of the hadrons. One is due to the background meson cloud which is represented by potential \(V = -(2/3)\alpha_s/r\) and resembles the crystal field on a crystal electron. For the external force we have considered an average harmonic oscillator type of force which is of confinement type. The potential may be expressed as:

\[ V_{ij} = -\frac{\alpha}{r} + (F_iF_j)(-\frac{1}{2}Kr^2) \]  

(4)

where the coupling constant \(\alpha = \frac{2}{3}\alpha_s\); \(F_iF_j = -\frac{2}{3}\) for qq interaction [12] and \(K\) is the strength parameter. So \(V_{ij}\) may be recast as:

\[ V_{ij} = -\frac{\alpha}{r} + ar^2 \]  

(5)

where \(a=K/3\). The ratio of the constituent mass and the effective mass of the diquark can be estimated by using the same formalism as in equation (3) in quasi particle approximation.
and can be expressed as:

\[
\frac{m}{m^*} = 1 + \frac{\alpha}{2ar^3}
\]  

(6)

Here \( m \) represents the normal constituent mass of the diquark and \( m^* \) is the effective mass of the diquark, \( r \) is the radius parameter of the diquark. To calculate the effective mass of the diquark we need the radius parameter \( r \) of the diquark which is not exactly known. We have inserted the radius of the \([ud]\) diquark as the effective radius of the \( \pi \) meson as \( r_{ud} = 5.38 \, GeV^{-1} \) as estimated in the context of the statistical model [13] fitting the charge radius of the pion. For the \([us]\) diquark we have used \( r_{us} = 6.06 \, GeV^{-1} \) fitting the experimental mass of the \( \Xi \) which has been supposed to be consisted of two \([us]\) diquark and one \( \pi \) in the framework of the quasi particle model of pentaquarks. With \( m_u = m_d = 360 \, MeV, m_s = 540 \, MeV \), \( \alpha = \frac{2}{3}\alpha_s = .393 \) as \( \alpha_s = 0.58 \) [14] and \( a = 0.003 \, GeV^3 \) estimated from the parameter \( K = 241.5 \, MeV fm^{-2} \) for the light quark system [15] the diquarks masses have been estimated as \( [m_{ud}] = 506 \, MeV \) and \( [m_{us}] = 700 \, MeV \). It may be mentioned that in QCD both gluon exchange interaction and instanton induced interaction favour spin singlet and colour antisymmetric diquark combination. The III model predicts the diquark mass as 420 MeV

**Magnetic Moment of multiquark states:-**

The magnetic moment of a system of particle due to its spin and orbital motion can be expressed as:

\[
\vec{\mu} = \sum_i \mu_i = \sum_i (g_i \vec{s}_i + \vec{l}_i) \mu_i
\]  

(7)

where \( g_i \) is the g factor of the \( i^{th} \) constituent, \( \mu_i \) is magneton of the constituent and equals to \( e_i/2m_i \). Considering diquark as the boson like particle with spin zero as in JW model [3], the magnetic moment for pentaquark system with two scalar diquark and one antiquark system can be written as [2],

\[
\vec{\mu} = \vec{l}_1 \mu_1 + \vec{l}_2 \mu_2 + g_3 \frac{\vec{T}}{2} \mu_3
\]  

(8)
It may be mentioned that the as diquarks are supposed to be in spin zero particles, the magnetic moment arises due to the relative momentum of the diquarks and the spin of the antiquark. Details of the corresponding wave function for diquarks have been discussed by Liu et al [2] in their excellent work on the investigation of the magnetic moments of pentaquarks in the context of the different models. The magnetic moment is very sensitive to the diquark mass. We have estimated the magnetic moment of the baryon antidecuplet members in the frame work of diquark-diquark-antiquark composition with input of the diquark masses estimated in the frame work of the quasi particle model of diquark from equation [5] as stated earlier. The results has been displayed in the Table-1. The results are compared with the results quoted in [2] in the frame work of JW model with two sets \( m_{ud}=720 \text{ MeV} \) \( m_{us}=900 \text{ MeV} \) (set-1)[16] and \( m_{ud}=420\text{MeV} \) , \( m_{us}=600\text{MeV} \) (set-2)[3]. Our results are found to be in agreement with the set-2 [3].

**Mass of the Antidecuplet States:-**

To estimate the mass of the antidecuplet in the frame work of the quasi particle diquark model we have considered the pentaquark configuration as in [3]. The diquark in the present model has been considered to be independent body behaving as quasi particle analogous to the many body systems which resembles the low lying excitation and as indicated earlier they are separate non interacting entities. Therefore as in the case of ideal gas of quasi particle, their energies may be assumed to be additive [17] and their effective mass characterises the dynamic properties of the quasi particle. Hence the mass of the pentaquarks can be represented as:

\[
M_x = 2M_D + M_\bar{q}
\]

where \( M_D \) is the mass of the diquark.

The baryon resonance states can be described as the exotic in the sense that they can be accommodated into the baryon antidecuplet as pentaquark states. Jaffe et al [3] has shown that \( \theta^+ \) baryon with positive parity lies in an ideally mixed \( SU(3)_f \times 10 \oplus 8_f \). They have argued that the Roper resonance fits to this classification naturally. \( \Sigma^{\frac{3}{2}}_+ \) multiplet mass
has been predicted to be 1750MeV in this framework. They have identified $N[ud]^2\bar{d}$ as Ropper resonance state with $N(1440)P_{11}$ and estimated the mass. The hyperons resonances $\Sigma$ has been suggested to be configuration as $[ud][su]d$ whereas $\Lambda$ as $[ud][ds]d$. They have also suggested $N_s[ud][su]s$ to be $N(1710)P_{11}$ state. We have estimated the mass of the antidecuplet member in the quasi particle model of diquark in the framework of pentaquark $q^4\bar{q}$ using the above equation. The results are displayed in Table-2 along with results of Jaffe et al [3]. The mass of the $\theta^+$ has been obtained well within the other predictions. The mass of the Ropper resonance as pentaquark state has been obtained as 1372MeV compared to experimental prediction 1440MeV [18]. $\Sigma$ mass has been obtained as 1566 MeV. PDG [18] has reported a bump at 1560MeV although a well established $\Sigma(1660)P_{11}$ and $\Lambda(1600)$ with $J^p=\frac{1}{2}^+$ have been reported. The $N_s$ is with same quantum number as nucleon and the mass is obtained in the current work as 1746MeV whereas PDG predicted $N(1710)P_{11}$ state. It may be mentioned that detailed information on nuclear resonances at the energy region is still awaited [6]. $\Sigma_s$ also suggested to have hidden strangeness. we have estimated the mass in the context of the quasi particle model as 1940MeV compared to 1880MeV by [18] although not well established.

Recently we have suggested diquark picture of nucleon (proton) considering it as diquark-quark system [19]. In the framework of the diquark picture the $\Delta$-$N$ mass difference may be attributed to the spin contribution of the scalar and axial vector diquark. The spin contribution runs as[20]:

$$ E_{SD} = \frac{8}{9} \frac{\alpha_s}{m_2m_2} \bar{s}_1 \cdot \bar{s}_2 |\psi(0)|^2 $$

where $|\psi(0)|^2$ is the ground state wave function of the diquark. For diquark we use the statistical model wave function as the trial wave function [13]. The mass of the scalar diquark in the context of the nucleon has been estimated as 272 MeV in our previous work [19]. The mass of the axial vector diquark has been estimated as 356 MeV with the input of $r_{ud}$ axial vector as 0.8fm as taken by Nagata et al [21]. The mass difference has been estimated considering the mass difference of the axial vector and scalar diquark and their
spin contribution. The result is obtained as:

\[ M_\Delta - M_N = 316 \text{MeV} \]  \hspace{1cm} (11)

The experimental prediction is \( M_\Delta - M_N \sim 300 \text{MeV} \)

**Conclusions:-**

In the present work we have investigated the magnetic moment and the mass of the members of the baryon antidecuplet in the frame work of the quasi particle model suggested by us. The magnetic moment is important quantity to be studied. Although different model yields different vales they in turn affect the photo production and electro production. The model is found to be reasonably successful in describing the properties of the baryons and exotics. In the current picture diquark is described as the quasi particle which embodies the interaction of the background medium. Like the quasi particle in the many body system its motion gets modified by the interaction of the back ground. They are behaving as free and independent entity. The situation is somewhat like the phonon particle we come across in the context of superfluidity in usual superfluid like He-II as suggested by Landau. They are hypothetical particle and do not have independent existence and each phonon creates an environment around itself so that they behave like free particle which prevents the condensation of He-II below 4.2\(^0\)K. The diquarks may be suggested to be floating like independent body in the medium which is superfluid. It is pertinent to point out here that a number of works are in the literature [22] which hints at the possibility of behaving the QCD vacuum as superfluid. It is interesting to observe that the simple addition of the masses of the diquarks and the anti quark reproduces the mass of the particle reasonably. It may be mentioned that the accurate measurement of properties of the baryon resonances experimentally is not complete [6] and the experimental results are somewhat uncertain. In the present model the diquark mass depends on the radius of the diquark which is not known exacly and may be supposed to be the main source of uncertainty. The spin contribution from scalar and axial vector diquark estimated in the current investigation is found to be
reasonably well with experimental predictions. So it may be asserted that consideration of diquark as the fundamental building block of nature may not be far from the reality. However the true nature of the diquark is yet to be settled. We would investigate the properties of other baryon and their resonances. The strange quark contribution to the properties of proton is an important candidate which would be investigated in our future works in framework of the current diquark model.
Table I: Magnetic Moment of AntiDecuplet States

<table>
<thead>
<tr>
<th>$Y, I, I_3$</th>
<th>Set – I</th>
<th>Set – II</th>
<th>Set – III</th>
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<td>0.219</td>
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<td>0.085</td>
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<tr>
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<td>0.12</td>
<td>0.1004</td>
</tr>
<tr>
<td>0, 1, -1</td>
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<td>0.09</td>
<td>0.116</td>
</tr>
<tr>
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<td>0.06</td>
<td>0.008</td>
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<td>-1, $\frac{3}{2}, -\frac{3}{2}$</td>
<td>0</td>
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<td>0</td>
</tr>
<tr>
<td>-1, $\frac{3}{2}, -\frac{1}{2}$</td>
<td>0.06</td>
<td>-0.06</td>
<td>-0.008</td>
</tr>
<tr>
<td>-1, $\frac{3}{2}, -\frac{3}{2}$</td>
<td>0.12</td>
<td>-0.12</td>
<td>-0.017</td>
</tr>
</tbody>
</table>

Set-I: with $m_{ud}=720$ MeV, $m_{us}=m_{ds}=900$ MeV from [16];
Set-II: with $m_{ud}=420$ MeV, $m_{us}=m_{ds}=600$ MeV from [3];
Set-III: Present Result with $m_{ud}=506$ MeV, $m_{us}=m_{ds}=700$ MeV

Table II: Masses of the Baryon Antidecuplet

<table>
<thead>
<tr>
<th>Particle</th>
<th>Results from JW Model[3]</th>
<th>Our Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta^+$</td>
<td>1540</td>
<td>1552</td>
</tr>
<tr>
<td>$N$</td>
<td>1440</td>
<td>1372</td>
</tr>
<tr>
<td>$\Sigma$</td>
<td>1660</td>
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<td>$\Lambda$</td>
<td>1600</td>
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<td>$\Sigma_s$</td>
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<td>1940</td>
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